Ch 35. Interference
35-1. Interference & Coherence Sources

\[ c = \lambda f \]
Interference

Path difference

\( \lambda/2, 3\lambda/2, 5\lambda/2 \ldots \)
\( 0, \lambda, 2\lambda, 3\lambda \ldots \)

Destructive interference
Constructive interference

Coherent Sources: two monochromatic sources of the same frequency & with any definite, constant phase relation (not necessarily in phase).
35-2. Two Source Interference of Light

Thomas Young’s experiment (1800)

Assumptions:
- Monochromatic
- Coherent

Path difference
\[ r_2 - r_1 = d \sin \theta \]

Constructive interference (Bright fringes):
\[ d \sin \theta = m\lambda \]

Destructive interference (Dark fringes):
\[ d \sin \theta = (m + \frac{1}{2})\lambda \]

\[ m = 0, \pm 1, \pm 2, \ldots \]
Interference Fringes

For small angles only

Constructive interference in Young’s Exp:

\[ y_{max} = R \tan \theta_m \approx R \sin \theta_m \]

\[ y_{max} = Rm \frac{\lambda}{d} \quad (R \gg d, R \gg y_m) \]

Center is a maximum

\[ y_{min} = R(m + 1/2) \frac{\lambda}{d} \]

\[ = \pm R \frac{\lambda}{2d}, \pm 3R \frac{\lambda}{2d}, \pm 5R \frac{\lambda}{2d} \ldots \]

Spacing between adjacent maxima /minima: \( R \frac{\lambda}{d} \)
35-3. Intensity in Interference Patterns:
Mathematical Method

Electric fields at point P:
\[ E_1 = E \cos (\omega t + \phi) \]
\[ E_2 = E \cos \omega t \]

Superposition:
\[ E_1 + E_2 = E \cos (\omega t + \phi) + E \cos \omega t \]
\[ = 2E \cos (\phi/2) \cos (\omega t + \phi/2) \]
\[ = E_P \cos (\omega t + \phi/2) \]

Amplitude:
\[ E_P = 2E |\cos (\phi/2)| \]

Intensity
\[ I \propto E_P^2 = 4E^2 \cos^2 (\phi/2) \]
Or:
\[ I = I_0 \cos^2 (\phi/2) \]
Intensity in Interference Patterns: Phasor Diagram

Amplitude: \( E_P = 2E \left| \cos \left( \frac{\phi}{2} \right) \right| \)

Intensity
\[ I \propto E_P^2 = 4E^2 \cos^2 \left( \frac{\phi}{2} \right) \]
Or: \[ I = I_o \cos^2 \left( \frac{\phi}{2} \right) \]
Intensity at $y$

$$\frac{\phi}{2\pi} = \frac{r_2 - r_1}{\lambda}$$

Phase difference: $$\phi = \frac{2\pi}{\lambda} (r_2 - r_1) = k(r_2 - r_1) = kd \sin \theta = \frac{2\pi d}{\lambda} \sin \theta$$

Intensity: $$I = I_o \cos^2 \left(\frac{\phi}{2}\right) = I_o \cos^2 \left(\frac{\pi d}{\lambda} \sin \theta\right)$$

Maximum Intensity: $$\frac{\pi d}{\lambda} \sin \theta = m\pi$$ or: $$d \sin \theta = m\lambda$$

Small slits ($y \ll R$, then $\sin \theta = y/R$) $$I = I_o \cos^2 \left(\frac{\pi dy}{\lambda R}\right)$$

All peaks have same intensity.