15-8. Normal Modes

A string with both ends fixed, length L, for standing wave to exist:

\[ L = n \frac{\lambda}{2} \quad n=1, 2, 3, \ldots \]

Standing wave wavelength

\[ \lambda_n = \frac{2L}{n} \]

Fundamental frequency

\[ f_1 = \frac{v}{\lambda_1} = \frac{v}{2L} \]

Harmonics, or overtones if \( n > 1 \)

\[ f_n = \frac{v}{\lambda_n} = n \frac{v}{2L} = nf_1 \]

\( n^{th} \) harmonic is \((n-1)^{th}\) overtone

Wave function

\[ y_n(x,t) = A_{SW} \sin k_n x \sin \omega_n t \]
Harmonics

Since \( v = \sqrt{\frac{F}{\mu}} \)

\[
f_1 = \frac{v}{2L} = \frac{1}{2L} \sqrt{\frac{F}{\mu}}
\]

for string fixed at both ends

Normal mode: a motion in which all particles of the system move sinusoidally with the same frequency.

Resonant frequency: frequency at which standing waves are produced

Multiple resonant frequencies / normal modes for a string

Single resonant frequency for a spring / pendulum