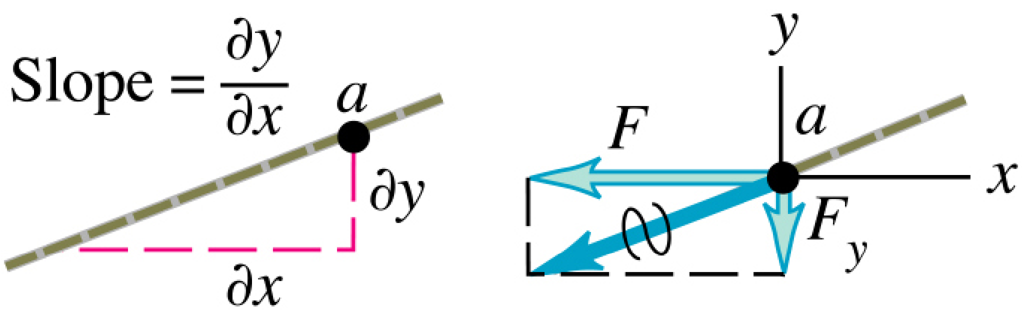
15-5. Energy in Wave Motion

1 dimensional: wave on a string



$$F_y(x,t) = -F\frac{\partial y(x,t)}{\partial x}$$

For any wave on a string, instantaneous rate of energy transfer

$$P(x,t) = F_y(x,t)v_y(x,t) = -F\frac{\partial y(x,t)}{\partial x}\frac{\partial y(x,t)}{\partial t}$$

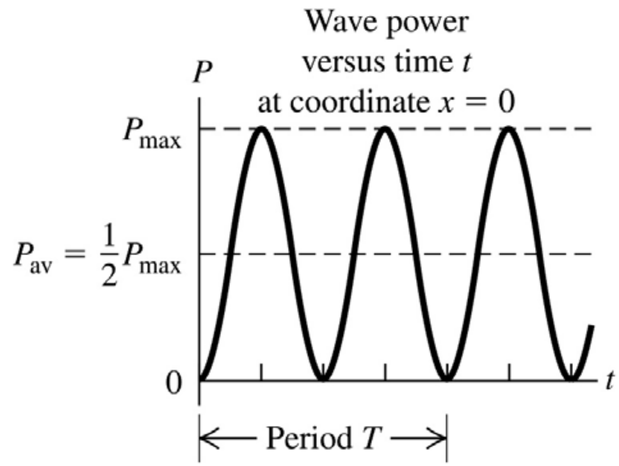
Energy Transferred by Sinusoidal Wave

For a sinusoidal wave

$$y(x,t) = A\cos(kx - \omega t)$$

$$P(x,t) = Fk\omega A^2 \sin^2(kx - \omega t)$$
$$= \frac{F\omega^2 A^2}{v} \sin^2(kx - \omega t)$$
$$= \frac{F\omega^2 A^2}{\sqrt{F/\mu}} \sin^2(kx - \omega t)$$

$$P(x,t) = \sqrt{\mu F}\,\omega^2 A^2 \sin^2(kx - \omega t)$$



$$P_{max} = \sqrt{\mu F}\,\omega^2 A^2 \qquad P_{av} = \frac{1}{2}\sqrt{\mu F}\,\omega^2 A^2$$

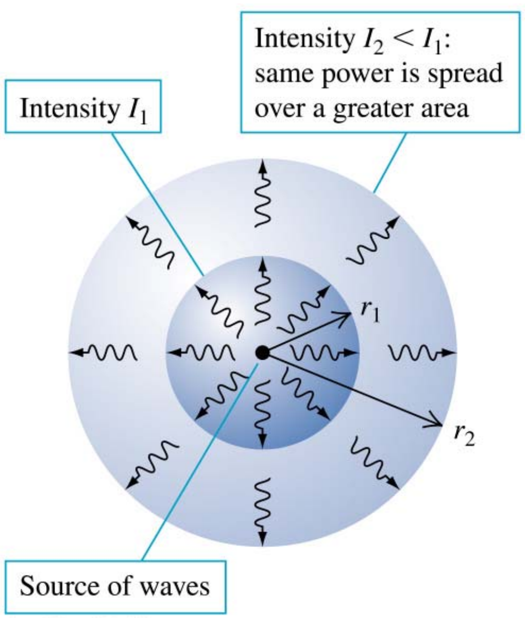
Energy Transported by Wave

3 dimensional wave

Power transported by wave per unit area, perpendicular to the propagation direction:

$$\text{Intensity I} = \frac{\text{energy/time}}{\text{area}} = \frac{\text{power}}{\text{area}} \propto A^2$$

Point source, isotropic medium - Spherical Wave

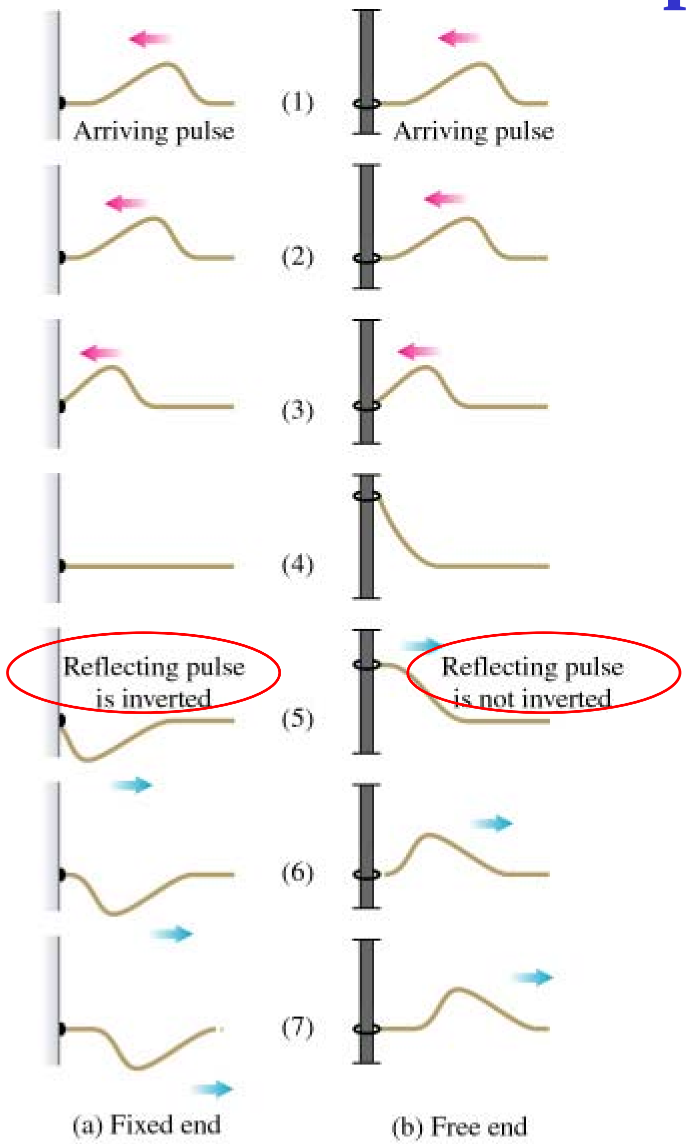


$$\text{I} = \frac{P}{4\pi r^2}$$

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$$

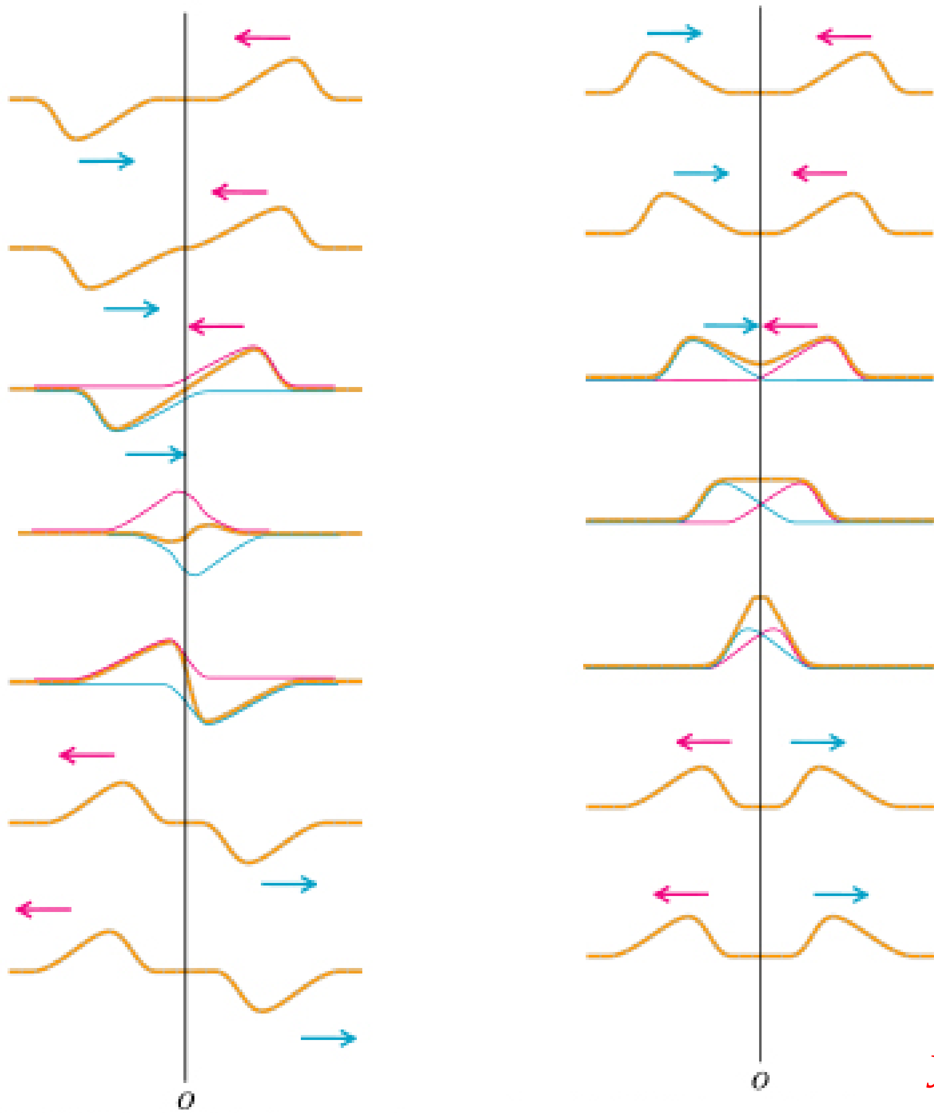
I=I_o at r, I at 2r?

15-6. Interference & Superposition



Interference & Superposition

Overlapping of waves

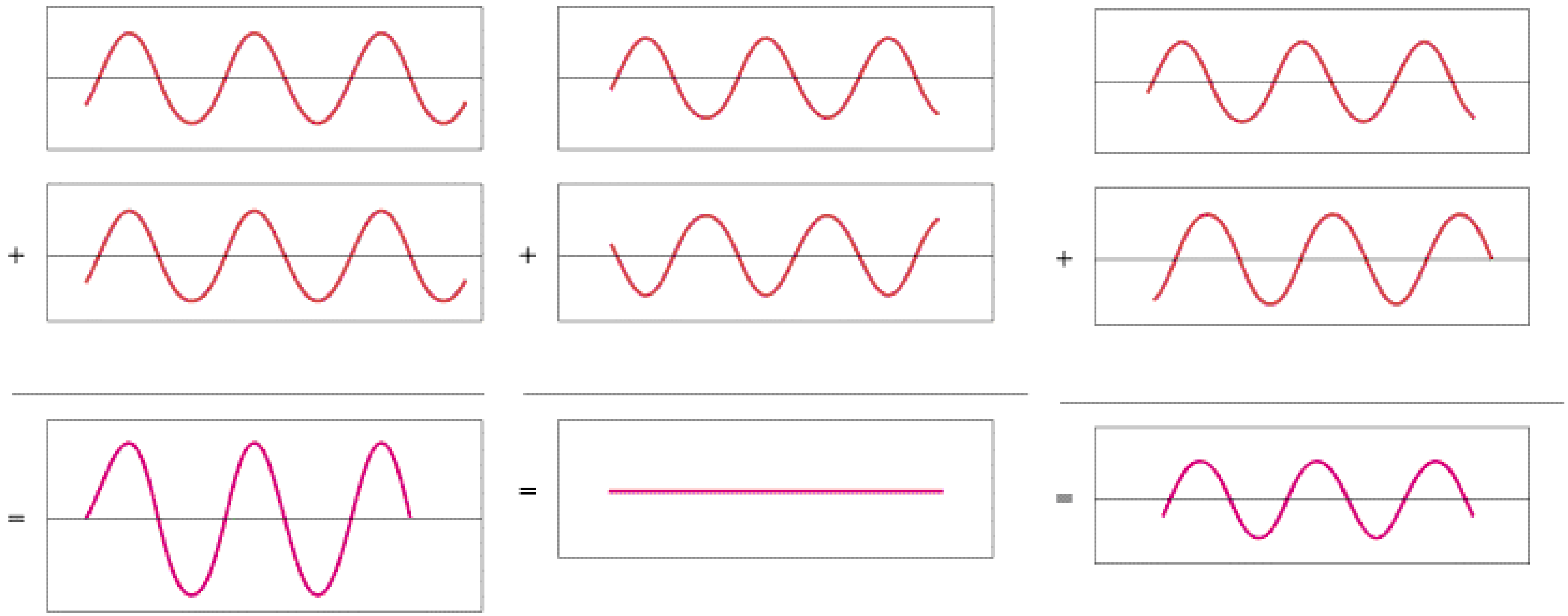


$y(x,t) = y_1(x,t)+y_2(x,t)$

Total displacement: Algebraic sum of the displacements in individual pulses

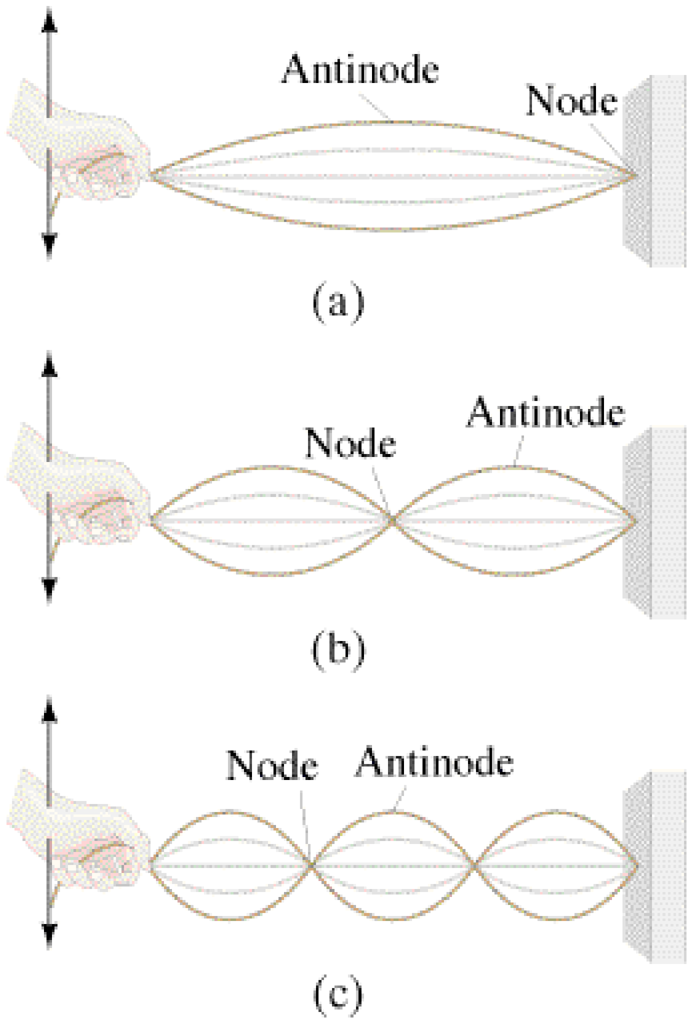
Interference of Continuous Waves

Constructive Destructive Partially Destructive



In Phase Completely out of phase Out of phase

15-7. Simple Example of Interference: Standing Waves



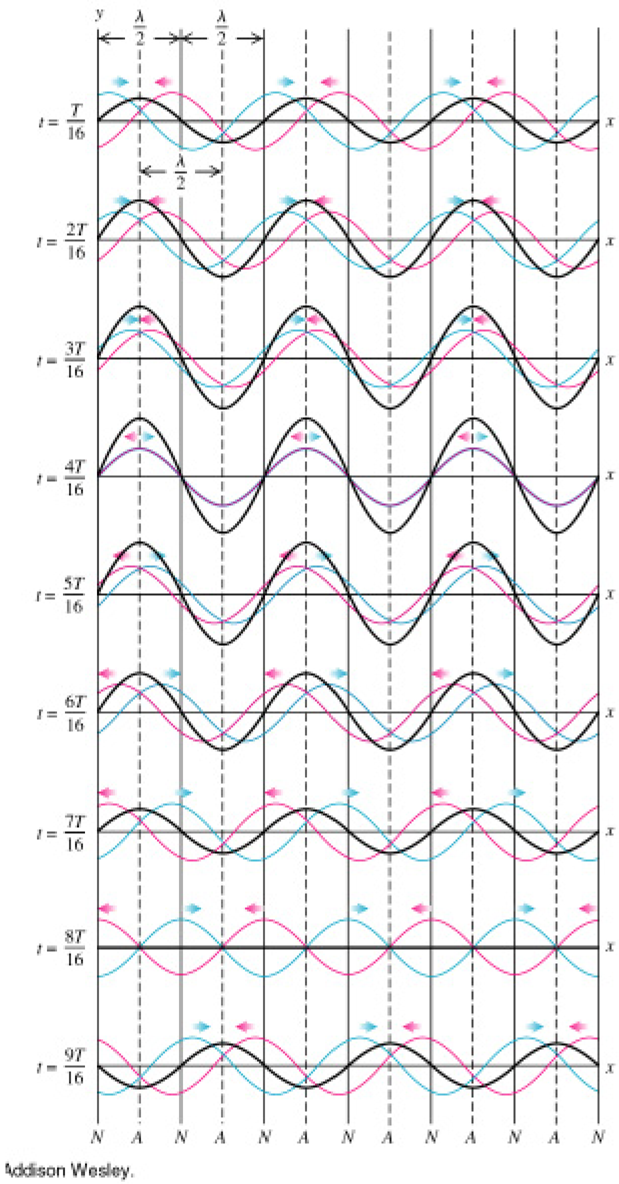
Nodes:

Points that never move

Antinodes:

Midway between nodes where the amplitude of motion is greatest

Does not transfer energy from one end to the other.



Standing Wave: Fixed at x=0

Red wave:
Incident wave traveling to the left, arriving at x=0
$y_1(x,t) = -A\cos(kx+\omega t)$

Blue wave:
Reflected wave traveling to the right from x=0
$y_2(x,t) = A\cos(kx-\omega t)$

"-" sign since the reflected wave from a fixed point is inverted

Mathematical Expression

$$y(x,t) = y_1(x,t) + y_2(x,t) = A[-\cos(kx+\omega t) + \cos(kx-\omega t)]$$

Since $\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$

Displacement

$$y(x,t) = A[-\cos kx \cos \omega t + \sin kx \sin \omega t + \cos kx \cos \omega t + \sin kx \sin \omega t]$$

$$= (2A \sin kx) \sin \omega t$$

$$= (A_{SW} \sin kx) \sin \omega t$$

Amplitude at given x: $|2A\sin kx|$

Maximum amplitude $A_{SW}=2A$

Positions of nodes: $\sin kx=0$, or $kx=n\pi$, n - integer

$x=n\lambda/2$ adjacent nodes $\lambda/2$ apart

Positions of antinodes: $\sin kx=+/-1$, or $kx=n\pi+\pi/2$

$x=n\lambda/2 \ +\lambda/4$

Characteristics of Standing Wave

Formed by two waves with	same frequency (ω, f, λ) same amplitude A opposite directions
Each point at position *x*	oscillates with ω, or $y \sim \sin\omega t$ fixed amplitude $\|2A\sin kx\|$ maximum amplitude at antinodes (2A) minimum amplitude at nodes (0)
Distance between	2 successive nodes (antinodes) is λ/2 a node and an adjacent antinode is λ/4

Every points between 2 successive nodes oscillate in phase

$P_{av} = 0$